Journal of Mechanical Science and Technology 24 (3) (2010) 801~804
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-0119-8

Identification of a crack in a beam by the boundary element method †

Jinhee Lee*

Dept. of Mechanical and Design Engineering, Hongik University, Chochiwon, Yeonki-gun, Choongnam, 339-701, Korea

(Manuscript Received July 20, 2009; Revised December 6, 2009; Accepted December 19, 2009)

Abstract

A method to detect a crack in a beam is presented. The crack is not modeled as a massless rotational spring, and the forward problem is solved for the natural frequencies using the boundary element method. The inverse problem is solved iteratively for the crack location and the crack size by the Newton-Raphson method. The present crack identification procedure is applied to the simulation cases which use the experimentally measured natural frequencies as inputs, and the detected crack parameters are in good agreements with the actual ones. The present method enables one to detect a crack in a beam without the help of the massless rotational spring model.

Keywords: Boundary element method; Crack detection; Inverse problem; Natural frequencies

1. Introduction

The crack identification in beams has been extensively investigated and many methods were proposed. The frequency contour plot method [1, 4-11] had been one of the most favored methods of crack detection in beams where the concept of the massless rotational spring model was crucial. Liang *et al.* [1] proposed the frequency contour method in which the crack was modeled as a massless rotational spring. The solution of a characteristic equation for a given natural frequency and a crack location gave the local stiffness. Because only one value of stiffness was permissible at a given crack location, the intersections of the various values of stiffness at various natural frequencies along the axial direction provided the location of the crack. Ostachowicz and Krawkczuk [2] and Dimarogonas and Paipetis [3], respectively, proposed the massless rotational spring models based on the fracture mechanics relations. The frequency contour method was also applied to the crack detection in stepped beams and truncated wedged beams [4-6]. Nandwana and Maiti [7] modeled the vibration of a beam in the presence of an inclined edge or internal normal crack using a rotational spring model. Lele and Maiti [8] extended the frequency contour plot method to the crack identification in beams based on the Timoshenko beam theory. Nikolakopoulos *et al.* [9] detected the crack location and the crack depth of frame structures by determining the superposed contour intersections. Hu and Liang [10] proposed two damage modeling techniques for the multiple crack detection. The continuum model was used first to identify the discretizing element of a structure that contained cracks, and then the spring damage model was used to quantify the location and size of the crack in each damaged element. Patil and Maiti [11] adopted the approach of Hu and Liang [10] and applied the transfer matrix method to the identification of multiple cracks.

Narkis [12] calculated the natural frequencies of a cracked simply supported beam by an approximate analytical solution and Morassi [13] derived an explicit expression of the frequency sensitivity to the crack. Shifrin and Ruotolo [14] proposed that $k+2$ equations were sufficient to form the system determinant for a beam with k cracks. Rizos *et al.* [15] estimated the crack size and location from the measured amplitudes of the structure vibrating at one of its natural modes and the analytical solution of the dynamic response. Ruotolo and Surace [16] developed a sensitivity approach to locate the damaged portion of the beam and utilized a massless rotational spring model. Lee [17, 18] applied the Newton-Raphson method to identify multiple cracks in a beam using the natural frequencies and the vibration amplitudes.

Owolabi *et al.* [19] used the changes in natural frequencies and frequency response function amplitudes as a function of crack depths and locations in the crack detection and estimated the crack depth and location based on the observed changes in the natural frequencies and mode shapes.

Most of the works in the crack detection in beams were based on the massless rotational spring model [1, 4-18] and few articles are available that do not depend on it. The objective of the present study is to present a procedure of identify-

† This paper was recommended for publication in revised form by Associate Editor Eung-Soo Shin
*Corresponding author. Tel.: +82 41 860 2589, Fax.: +82 41 862 2664
E-mail address: jinhlee@hongik.ac.kr

ing a crack in a beam that does not require the massless rotational spring model, and the boundary element method is used to compute the natural frequencies of the cracked beam.

The boundary element method has been successfully applied to many areas such as fracture mechanics and contact problems. In the boundary element method the governing differential equations are transformed into integral identities which are applicable over the boundary only. Because all the computations are restricted to the boundary it is particularly useful when subsequent changes in the meshes are made, which makes it ideal for the crack identification problem that requires the re-meshing of the cracked beam many times. Ali and Rajakumar [20] developed boundary element algorithms for vibration and a complete list of FORTRAN code for the boundary element analysis of vibration problems can be found in Ameen [21].

Table 1. Comparison of the natural frequencies of a simply supported beam without crack (h =0.0254 m, L =0.65 m, E =70 GPa, ρ =2696 kg/m^3 and ν =0.35).

	f_1^{NC} (Hz)	f_2^{NC} (Hz)
ANSYS (1141 bi-quadratic elements)	138.54	549.83
present study (104 quadratic elements)	138.63	549.37

Table 2. Ratios of the natural frequency of a beam with a crack to that without crack (simply supported boundary conditions, h =0.0254 m, L =0.65 m, E =70 GPa, ρ =2696 kg/m^3 and ν =0.35).

		β =0.1875		β =0.4375	
		$\frac{f_1}{f_1^{NC}}$	$\frac{f_2}{f_2^{NC}}$	$\frac{f_1}{f_1^{NC}}$	$\frac{f_2}{f_2^{NC}}$
Experiments [19]	α =0.1	0.9980	0.9962	0.9960	0.9994
	α =0.2	0.9956	0.9889	0.9849	0.9976
	α =0.3	0.9881	0.9712	0.9686	0.9952
	α =0.4	0.9781	0.9481	0.9418	0.9918
Present study (BEM)	α =0.1	0.9984	0.9956	0.9942	0.9992
	α =0.2	0.9947	0.9855	0.9840	0.9976
	α =0.3	0.9879	0.9679	0.9647	0.9947
	α =0.4	0.9777	0.9431	0.9371	0.9906

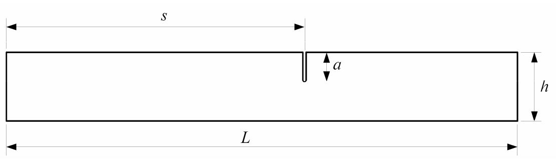
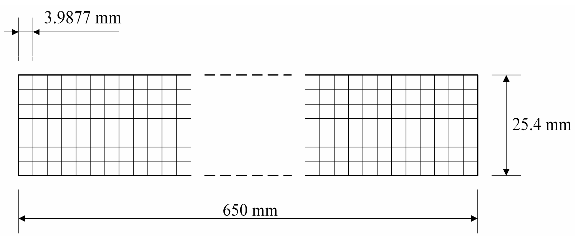
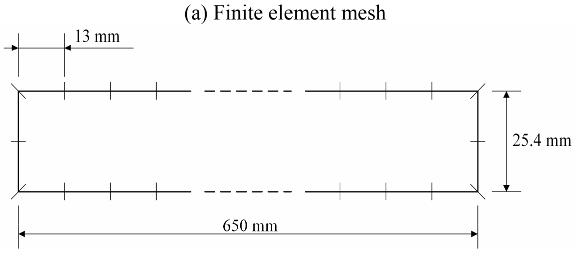


Fig. 1. Schematic geometry of a beam with a crack.



(a) Finite element mesh



(b) Boundary element mesh

Fig. 2. Finite element mesh and boundary element mesh of the beam without crack (| : element boundary).

2. Forward problem

The schematic geometry of a beam with a crack is given in Fig. 1. Parameters $\alpha = a/h$ and $\beta = s/L$ denote the normalized crack size and the normalized crack location where h and L are the thickness and the length of the beam.

Owolabi *et al*. [19] conducted experimental tests on simply supported beams with and without a crack and measured the natural frequencies. The beams were made of aluminum and their dimensions were $0.0254 \times 0.0254 \times 0.65$ m^3. Young's modulus, the density and Poisson's ratio were E =70 GPa, ρ =2696 kg/m^3 and ν =0.35, respectively. The cracks had been obtained by a thin saw cut and the width of the crack was around 0.4 mm.

The program of Ameen [21] is used to compute the natural frequencies of a beam without a crack. The natural frequencies of a beam without crack f_i^{NC} $(i = 1, 2)$ computed by the boundary element method, as well as the results of commercially available finite element program ANSYS [22], are given in Table 1. The schematics of the finite element mesh (1141 bi-quadratic elements) and the boundary element mesh (104 quadratic elements) are shown in Fig. 2. The natural frequencies computed by the boundary element method are practically identical to those by ANSYS, which confirms that the boundary element method used in the present study is accurate and reliable.

The program of Ameen [21] is also used to compute the natural frequencies of a beam with a crack. The number of quadratic elements of the boundary element mesh is 168, and the nodes are densely populated in the area near the crack tip. In modeling fracture problems the main difficulty arises from the presence of an infinite stress at the crack tip. To overcome this intricacy Becker and Fenner [23] introduced singularity elements in their boundary element analysis of fracture me-

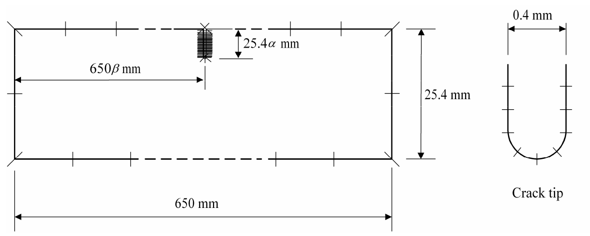


Fig. 3. Boundary element mesh of the beam and enlarged mesh around the crack tip.

chanics. The cracks of Owolabi *et al*. [19], however, were not obtained as the result of the fracture activity and the crack tips in the present study are modeled as a round tip as shown in Fig. 3. The ratios of the natural frequencies of a beam with a crack to those without crack f_i/f_i^{NC} $(i=1,\ 2)$ are listed in Table 2 where the experimental results of Owolabi *et al*. [19] are given for comparison.

In the real situation, the crack might not be so big as considered in the present study. In many cases the crack surface is almost invisible with the width less than 0.001 mm. However, the sizes of the crack in Table 2 are chosen to be in the range of α =0.1~0.4 to meet those of the experimental measurements [19].

3. Inverse problem

It is assumed that the measurements of the lowest two natural frequencies f_1^m and f_2^m are available. For the identification of a single crack there are two unknown crack parameters α and β. The Newton-Raphson procedure is applied as follows:

(a) assume the initial values of α and β,

(b) re-mesh the boundary element mesh according to the new crack position $L\beta$ and crack size $h\alpha$,

(c) solve the forward problem for the natural frequencies f_1 and f_2 with the updated boundary element mesh, and evaluate the Jacobian matrix $[J]$

$$[J]=\begin{bmatrix}\dfrac{\partial f_1}{\partial\alpha} & \dfrac{\partial f_1}{\partial\beta}\\ \dfrac{\partial f_2}{\partial\alpha} & \dfrac{\partial f_2}{\partial\beta}\end{bmatrix} \tag{1}$$

and the residuals

$$\begin{cases}\Re_1 = f_1 - f_1^m\\ \Re_2 = f_2 - f_2^m\end{cases} \tag{2}$$

(d) Solve the equation

$$[J]\begin{Bmatrix}d\alpha\\ d\beta\end{Bmatrix} = -\begin{Bmatrix}\Re_1\\ \Re_2\end{Bmatrix} \tag{3}$$

for $\{d\alpha \quad d\beta\}^T$,

(e) update the crack parameters

$$\alpha_{\text{new}} = \alpha_{\text{old}} + d\alpha,\ \ \beta_{\text{new}} = \beta_{\text{old}} + d\beta \tag{4}$$

(f) iterate the procedure (b)-(e) until the residuals become sufficiently small.

The elements of the Jacobian matrix are the sensitivities of the natural frequencies with respect to the crack parameters and they are computed numerically. For example, $\partial f_1/\partial\alpha$ is computed by

Table 3. Numerical simulations of the crack identification procedure.

	Actual		Experiments [19]		Detected	
	α	β	$\frac{f_1}{f_1^{NC}}$	$\frac{f_2}{f_2^{NC}}$	α	β
Case A	0.3	0.1875	0.9881	0.9712	0.2755	0.2072
Case B	0.2	0.4375	0.9849	0.9976	0.1930	0.4361

$$\frac{\partial f_1}{\partial\alpha} = \frac{f_1(\alpha+\delta,\beta) - f_1(\alpha,\beta)}{\delta},\quad (|\delta| \ll 1) \tag{5}$$

The forward problem is solved three times per iteration to build the Jacobian matrix and the residuals. To avoid overshoots in the early stage an underrelaxation is performed during the first two or three iterations

$$\alpha_{\text{new}} = \alpha_{\text{old}} + 0.25\ d\alpha,\quad \beta_{\text{new}} = \beta_{\text{old}} + 0.25\ d\beta \tag{6}$$

The crack identification procedure is applied to the two simulation cases shown in Table 3. In case A, where the actual crack parameters are α =0.3 and β =0.1875, the natural frequencies f_1^m =136.98 Hz and f_2^m =533.55 Hz from the experiments [19] and f_i^{NC} $(i=1,\ 2)$ are input as the measurements. When the procedure (a)- (f) is applied the detected crack parameters are found to be α =0.2755 and β =0.2072. The observed error of the normalized crack location is within 0.02 and the error of the estimated crack sizes is within 8.2 percent of the actual size. In case B, where the actual crack parameters are α = 0.2 and β =0.4375, the natural frequencies f_1^m =136.54 Hz and f_2^m =548.05 Hz from the experiments [19] and f_i^{NC} $(i=1,\ 2)$ are input as the measurements and the detected crack parameters are α = 0.1939 and β =0.4361, which are in excellent agreements with the actual ones.

Proper selection of the initial guesses of the crack parameters is important because the present method is based on the Newton-Raphson iteration method. The ranges of the initial guesses to produce a converged solution vary from case to case. In the simulation case A, the convergence of the solution is achieved when the initial guess of crack location is in the range of 0.11< β <0.46 while the initial guess of crack size is α =0.1. Also when the initial guess of crack size is α =0.2, the range of the successful initial guess of crack location is in the range of 0.09< β <0.49. It indicates that the convergence of the solution is not strongly affected by the variations of the initial guess of the crack size.

4. Conclusions

A method to detect a crack in a beam where the crack is not modeled as a massless rotational spring is presented. The forward problem is solved for the natural frequencies of beams with and without a crack using the boundary element method. The inverse problem is solved iteratively for the crack location and the crack size by the Newton-Raphson method. The present method is applied to the simulation cases which use the experimentally measured natural frequencies as inputs, and the detected crack parameters are in good agreements with the actual ones. It shows one can detect a crack in a beam without the help of the massless rotational spring model that represents the crack. The ranges of the initial guesses to produce converged solutions are investigated and it is found that the convergence of the solution is less affected by the variations of the initial guess of the crack size than that of the crack position.

Acknowledgements

This work was supported by 2009 Hongik University Research Fund.

References

[1] R. Y. Liang, F. K. Choy and J. Hu, Detection of cracks in beam structures using measurements of natural frequencies, *Journal of the Franklin Institute*, 328 (4) (1991) 505-518.
[2] W. M. Ostachowicz and M. Krawkczuk, Analysis of the effect of cracks on the natural frequencies of a cantilever beam, *Journal of Sound and Vibration*, 150 (2) (1991) 191-201.
[3] A. D. Dimarogonas and S. A. Paipetis, *Analytical Methods in Rotor Dynamics*, London, Elsevier Applied Science, (1983).
[4] B. P. Nandwana and S. K. Maiti, Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies, *Journal of Sound and Vibration*, 203 (3) (1997) 435-446.
[5] T. D. Chaudhari and S. K. Maiti, A study of vibration of geometrically segmented beams with and without crack, *International Journal of Solids and Structures*, 37 (5) (2000) 761-779.
[6] S. Chinchalkar, Determination of crack location in beams using natural frequencies, *Journal of Sound and Vibration*, 247 (3) (2001) 417-429.
[7] B. P. Nandwana and S. K. Maiti, Modelling of vibration of beam in presence of inclined edge or internal crack for its possible detection based on frequency measurements, *Engineering Fracture Mechanics*, 58 (3) (1997) 193-205.
[8] S. P. Lele and S. K. Maiti, Modelling of transverse vibration of short beams for crack detection and measurement of crack extension, *Journal of Sound and Vibration*, 257 (3) (2002) 559-583.
[9] P. G. Nikolakopoulos, D. E. Katsareas and C. A. Papadopoulos, Crack identification in frame structures, *Computers & Structures*, 64 (1-4) (1997) 389-406.
[10] J. Hu and R. Y. Liang, An integrated approach to detection of cracks using vibration characteristics, *Journal of the Franklin Institute*, 330 (5) (1993) 841-853.
[11] D. P. Patil and S. K. Maiti, Detection of multiple cracks using frequency measurements, *Engineering Fracture Mechanics*, 70 (12) (2003) 1553-1572.
[12] Y. Narkis, Identification of crack location in vibrating simply supported beams, *Journal of Sound and Vibration*, 172 (4) (1994) 549-558.
[13] A. Morassi, Identification of a crack in a rod based on changes in a pair of natural frequencies, *Journal of Sound and Vibration*, 242 (4) (2001) 577-596.
[14] E. I. Shifrin and R. Ruotolo, Natural frequencies of a beam with an arbitrary number of cracks, *Journal of Sound and Vibration*, 222 (3) (1999) 409-423.
[15] R. F. Rizos, N. Aspragathos and A. D. Dimarogonas, Identification of crack location and magnitude in a cantilever beam from the vibration modes, *Journal of Sound and Vibration*, 138 (3) (1990) 381-388.
[16] R. Ruotolo and C. Surace, Damage assessment of multiple cracked beams: numerical results and experimental validation, *Journal of Sound and Vibration*, 206 (4) (1997) 567-588.
[17] J. Lee, Identification of multiple cracks using natural frequencies, *Journal of Sound and Vibration*, 320 (3) (2009) 482-490.
[18] J. Lee, Identification of multiple cracks using vibration amplitudes, to be published in *Journal of Sound and Vibration*.
[19] G. M. Owolabi, A. S. J. Swamidas and R. Seshadri, Crack detection in beams using changes in frequencies and amplitudes of frequency response functions, *Journal of Sound and Vibration*, 265 (1) (2003) 1-22.
[20] A. Ali and C. Rajakumar, *The boundary element method: Applications in sound and vibration*, Taylor & Francis, (2004).
[21] M. Ameen, *Boundary Element Analysis, Theory and Programming*, Narosa Publishing, (2001).
[22] K. L. Lawrence, *ANSYS tutorial: release 10*, SDC Publications, (2006).
[23] A. A. Becker and R. T. Fenner, Axisymmetric fracture mechanics analysis by the boundary integral equation method, *International Journal of Pressure Vessels and Piping*, 18 (1) (1985) 55-75.

Jinhee Lee received B.S. and M.S. degrees from Seoul National University and KAIST in 1982 and 1984, respectively. He received his Ph.D. degree from University of Michigan, Ann Arbor in 1992 and joined Dept. of Mechanical and Design Engineering of Hongik University in Choongnam, Korea. His research interests include inverse problems, pseudospectral method, vibration and dynamic systems.